Exercise 1:

A new robotic arm is tested prior to being marketed to the general public. The test results on 50 independent samples are shown below:

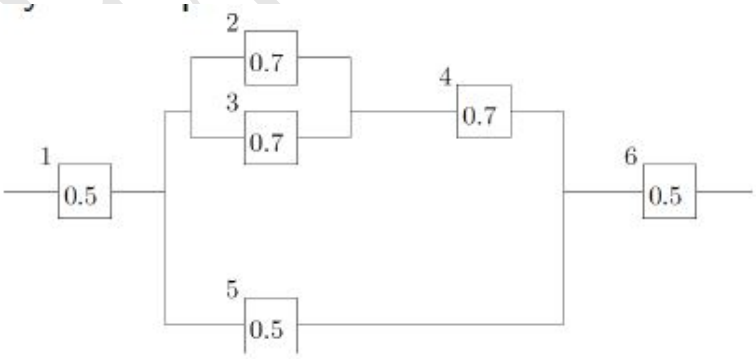
	Mechanical Defect	No Mechanical Defect
Electrical Defect	5	10
No Electrical Defect	5	30

Assume that these relative proportions are perfectly representative of the product (and of its flaws) outside the sample.

1. What is the probability that a robotic arm has a mechanical defect?
2. What is the probability that a robotic arm has neither a mechanical nor an electrical defect?
3. What is the probability that a robotic arm has a mechanical defect if it is known that it has an electrical defect?
4. What is the probability that each of 3 robotic arms purchased by a company has at least one defect (assume that defects occur independently)?

Exercise 2:

Consider the following system with six components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown. Assume that the components function or fail independently. What is the probability that the system operates?



Exercise 3:

Amongst all vehicles that have an ignition problem, assume that

- the starter is at fault in 50% of the cases;
- the battery is at fault in 40% of the cases, and
- one of the plugs is at fault the rest of the time.

Furthermore, amongst all vehicles with an ignition problem, the proportion of vehicles of make X is

- 10% in cases where the starter is at fault;
- 20% in cases where the battery is at fault, and
- 5% in cases where one of the plugs is at fault.

A vehicle of make X is brought to a mechanic because of an ignition problem. What should be the first line of attack for the mechanic: starter, battery, or plugs?

Exercise 4:

Consider an ordinary 52-card North American playing deck (4 suits, 13 cards in each suit).

1. How many different 5−card poker hands can be drawn from the deck?
2. How many different 13−card bridge hands can be drawn from the deck?
3. What is the probability of an all-spade 5−card poker hand?
4. What is the probability of a flush (5−cards from the same suit)?
5. What is the probability that a 5−card poker hand contains exactly 3 Kings and 2 Queens?
6. What is the probability that a 5−card poker hand contains exactly 2 Kings, 2 Queens, and 1 Jack?
7. The deck is shuffled. Four cards are then drawn from. What is the probability of having drawn a spade, a heart, a diamond, and a club, in that order?

Exercise 5:

Let X be a random variable whose probabilitic behaviour is

$$f(x) = \frac{2x+1}{25}, \quad x = 0, 1, 2, 3, 4.$$

1. Show that f defines the probability mass function of X. Graph the bar chart and probability histogram of X.
2. Find and graph the cumulative distribution function of X.
3. Find the expectation and the variance of X.
4. Find the expectation and the variance of $Y = 2/3X - \sqrt{2}$.