

Exercise 1:

The joint probability mass function of (X, Y) is given by:

| $X \setminus Y$ | 0 | 1 | 2 |
|-----------------|------|------|-----|
| -1 | a | $2a$ | a |
| 0 | 0 | a | a |
| 1 | $3a$ | 0 | a |

1. Determine the constant a .
2. Find the marginal probability mass functions of X and Y .
3. Are X and Y independent random variables?
4. Compute $V(2X + 3Y)$ and $Cov(2X, 5Y)$.
5. Calculate $E(Y|X = -1)$ and $V(Y|X = -1)$.
6. Compute $P(X \geq -1|Y = 2)$
7. Find the probability mass function of $U = XY$.

Exercise 2:

The percentage of defects in an assembly line is 0.006. Among 100 items chosen at random, determine the probability that there is exactly one defective item, at least one defective item. Calculate these two probabilities using the Poisson approximation and comment on your results.

Exercise 3:

In a batch of 25 TVs, 5 of which are broken down, 8 TVs are randomly selected. What is the probability that among these 8 TVs, at least 2 are broken down? Determine the expectation and the standard deviation of the broken down TVs among the 8 TVs selected. If we had a batch of 100 machines instead of 25, would the binomial distribution give a good approximation?

Exercise 4:

An electrical company is conducting interviews to fill 2 open positions. The probability that any person interviewed will meet the qualifications and accept a position is 0.8. What is the probability that this company would have to conduct exactly 4 interviews to fill the 2 positions? What is the probability that this company would have to conduct fewer than 4 interviews?

Exercise 5:

Revenue Canada estimates that the probability of a citizen completing his or her tax return correctly is 0.26; the probability of making errors that benefit the citizen is 0.45; the probability of making errors that benefit the government is 0.15; and the probability of making both types of errors is 0.14. If 15 tax returns are randomly selected, what is the probability of finding 6 correctly filled out, 3 containing errors that benefit the citizen, 4 containing errors that benefit the government, and 2 containing both types of errors? If two thousand tax returns are selected at random, how many can we expect to find that are correctly filled out?